

ISLAMIC GOVERNANCE AND THEOCRACY

By

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Allah is Sovereign. Islamic political system is based on its specific worldview that is essential to know in any understanding of Islam. The Qur'an tells us that Allah is the Creator and Lord of the whole universe including humankind and all that is associated with them. He is overpowering and is irresistibly dominant over all His creation. He knows all and governs all. He is ever living and everlasting and all His creation, willingly or unwillingly, is obedient to Him. Whatever He wills gets done. It is His power that is established and none can interfere in it in anyway. Thus it is Allah who possesses all the powers and attributes of sovereignty and none else

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$$\exists u \forall v \forall x \exists y p(f(u), v, x, y) \rightarrow q(u, v, y)$$

the existentially quantified variable u is not within the scope of the universally quantified variable, v , and x , and hence the function $f(u)$ can be replaced by $f(a)$ with “ a ” being a constant, and the existential quantifier $\exists u$ is removed from this expression. Thus the new expression after this type of skolemization is,

$$\forall v \forall x \exists y p(f(a), v, x, y) \Rightarrow q(a, v, y)$$

Having understood what is skolemization and applying this algorithm to the expression (3), the skolemized expression becomes as,

$$\forall y (\sim a(f(a), y, g(y)) \vee (b(a, h(y)) \wedge c(y) k(y)))$$

Step 5: Remove all universal quantifiers, since the universally quantified variables are implicitly retained in the expression.

In the event of this step the above expression may now be written as

$$(\sim a(f(a), y, g(y)) \vee (b(a, h(y)) \wedge c(y), k(y)))$$

Step 4: Purge existential quantifiers. All existentially quantified variables should be replaced by skolem functions, and the corresponding existential quantifiers should be removed.

The skolemization may be understood as follows. If there are existential quantifiers

which are preceded by one or more universal quantifiers, i.e. the existential

quantifiers are within the scope of universal quantifiers then replace all the

existentially quantified variable by a function symbol not appearing anywhere in

the expression. For example, in the expression

$$\forall v \forall x \exists y p(v, x, y) \rightarrow q(v, y)$$

the existential quantifier $\exists y$ is within the scope of the universal quantifiers $\forall v$ and

$\forall x$, so according to this algorithm, the skolemized expression is

$$\forall v \forall x p(v, x, g(v, x)) \rightarrow q(v, h(v, x))$$

This expression is obtained by replacing variable y , which is existentially quantified

by operator $\exists y$ in the earlier expression, by the function $g(v, x)$ in left hand side and

by $h(v, x)$ in the right hand side of the expression.

Another type of skolemization is that if the existential quantifier does not come

within the scope of the universal quantifier then there will not be any functional

dependency of existentially quantified variable on universally quantified variable,

and hence the existentially quantified variable can be replaced by a constant symbol.

For example, in the following expression

$(\forall x) (\text{soul}(x) \rightarrow \text{mortal}(x))$

$(\forall x) (\exists y) (\text{likes}(x,y))$

are in prenex form where as

$(\forall x)(\text{father}(x) \rightarrow (\exists y)(\text{son}(y) \wedge \text{loves}(x,y)))$

is not)

- Step 2: Drive in all negations immediately before an atom. Use for this purpose $\neg p$ instead of $(\neg p)$ and also use De Morgan's law, i.e., $(\exists x) \neg p(x)$ in place of $\neg(\forall x) p(x)$ and $(\forall x) \neg p(x)$ in place of $\neg(\exists x) p(x)$.

Using these algorithms the expression

$\exists x \forall y (\sim \forall z a(f(x), y, z) \vee (\exists u b(x, u) \wedge \exists v c(y, v)))$

is modified as

$\exists x \forall y (\exists z) \sim a(f(x), y, z) \vee (\exists u b(x, u) \wedge \exists v c(y, v))$

- Step 3: Rename variables, if necessary, so that all quantifiers have different variable assignments. It should be noted here that the renaming will not change the meaning of the formula because these variables just act as dummies for the corresponding quantifiers.

If we have an expression like

$\forall x (\sim p(x)) \vee (\exists y) (q(x, y)) \wedge (\forall x) p(x) \vee (\forall y) (\sim q(x, y))$

then according to the algorithm of step 3, $\forall x$ quantifier which is in the left most position is retained as it is whereas $(\forall x) (p(x) \vee (\forall y) (\sim q(x, y)))$ is replaced by the expression $(\forall z) (p(z) \vee (\forall w) (\sim q(z, w)))$.

28. RESOLUTION IN PREDICATE LOGIC

The resolution method in predicate logic precedes much as for the propositional logic. Again the stages are:

1. Form the conflict set (premises + negation of conclusion)
2. Convert the conflict set to a set of formulae in clause form
3. Repeatedly apply the *resolution rule* to try to derive a contradiction.
4. If a contradiction is found then the argument is valid.

But there are following two additional tasks that are needed to perform during the resolution procedure.

1. Eliminating existential quantifier and replacing the corresponding variable by either a constant (called a *Skolem constant*) or a function (called a *Skolem function*). This process is called *Skolemization*
2. The resolution rule "step 3", when modified to handle clause form formulae containing variables, requires an extra operation called "unification".

The steps for converting a given sentence into clause form may be described as follows:

Step 1: Convert to *prenex form*

(Note: a formula in the predicate logic in which all the quantifiers are at the front (i.e. have the whole formula with in their scope) is said to be in *prenex form*. For example.

property that the subject of the statement can have. We can denote the propositional function “x is sahabī” by sahabī (x). In logic this claim is written in short as “sahābī” and is known as “predicate” where as x is known as object and sahabī(x) is known as the predicate function. We can replace x with any element that belongs to the domain set A. If we replace x with Hazrat Abu Bakr i.e. Sahābī (Hazrat Abu Bakr) then it becomes a statement and the statement is true. On the other hand, if we replace x with Hazrat Umar-bin-Abdul Abu then the statement Sahābī(Hazrat Umar-bin-Abdul Azīz) is a false statement. The open sentence “Zarrar is a male” can be written in predicate form as male(zarrar).

Similarly, an open sentence with two or three variables can be written in predicate form as:

- i. offers (hanzala, prayer)
i.e., Hanzala offers prayer
- ii. keeps (khawla, fast)
i.e., Khawla keeps fast
- iii. obeys (fārābī, allah)
i.e., Fārābī obeys Allah
- iv. muallim (talha, saeed, waqas)
i.e., Talha is the muallim of Saeed and Waqas, etc.

If we want to write “all of the members of the set B is a sahabī” in predicate form then we have to use universal quantifier. The statement will be:

$$(\forall x \in B) (\text{sahābī} (x))$$

Similarly, if we want to write “Some of the members of the set B is ashra-e-mubashshirah”, then the statement will be:

$$(\exists x \in B) (\text{ashra-e-mubashshirah} (x))$$

- p: It is a month of Ramazan
- q: Fast is obligatory on 'A'
- r: 'A' keeps fast
- s: 'A' is sick

To apply the resolution procedure, we perform the following:

1. The conflict set of this argument is:
 $\{p \rightarrow q, (q \wedge \sim s) \rightarrow r, p, \sim s, \sim r\}$
2. Since $p \rightarrow q$ is equivalent to $\sim p \vee q$ and $(q \wedge \sim s) \rightarrow r$ is equivalent to $\sim q \vee s \vee r$, so the conflict set in clause form is:
 $\{\sim p \vee q, \sim q \vee s \vee r, p, \sim s, \sim r\}$
3. We then apply resolution to derive a contradiction:

S

i.	$\sim p \vee q$	Conflict Set
ii.	$\sim q \vee s \vee r$	
iii.	p	
iv.	$\sim s$	
v.	$\sim r$	
vi.	$\sim q \vee s$	From 2 and 5 by resolution
vii.	$\sim q$	From 4 and 6 by resolution
viii.	$\sim p$	From 1 and 7 by resolution
ix.	Contradiction	From 3 and 8 by resolution

4. We have found a contradiction in the conflict set, and so the argument is valid.

27. PREDICATE

The propositional function (explained in section 22)

$p(x)$: x is sahabī

has two parts. The first part, the variable x, is the subject of the propositional function. The second part - the claim "is sahabī"- refers to a

26. RESOLUTION IN PROPOSITIONAL LOGIC

A proof theory is a technique for establishing the validity of arguments. Although, two methods are already discussed, the third one, given below, is the most efficient

This method is as follows:

1. Form the *conflict set* (premises + negation of conclusion)
2. Convert the conflict set to a set of formulae in *clause form*
 (Note: A *literal* is a proposition letter or a proposition letter prefixed by \sim .
 Thus $b, c, \sim d$ are all literals; $a \vee b, a \wedge b$ and $\sim \sim a$ are not literal.
 A formula is in clause form if it is a literal or a collection of literals all joined by \vee .
 Thus $\sim p, p \vee q, \sim p \vee \sim q \vee r$ are all in clause form; $p \wedge q, p \rightarrow q$ and $\sim \sim p$ are not.)
3. Repeatedly apply the *resolution rule* described below to try to derive a contradiction.
4. If a contradiction is found then the argument is valid.

Consider the following argument:

If it is a month of Ramazan then fast is obligatory on 'A'

If fast is obligatory on 'A' and 'A' is not sick then 'A' keeps fast

It is a month of Ramazan

'A' is not sick

Therefore 'A' keeps fast

Symbolically

$p \rightarrow q, (q \wedge \sim s) \rightarrow r, p, \sim s \vdash r$
 where

Furthermore, if $p(x)$ denotes “ x is mortal”, then the above can be written as:

$$\sim(\forall x \in S) p(x) \equiv (\exists x \in S) \sim p(x)$$

Whatever be the proposition $p(x)$ the above relation is true i.e. “it is not true that for all x belongs to S , $p(x)$ is true” is equivalent to “for some x belongs to S , the negation of $p(x)$ is true”.

Similarly

$$\sim(\exists x \in S) p(x) \equiv (\forall x \in S) \sim p(x)$$

is true in general i.e.

“It is not true that for some x belongs to S , $p(x)$ is true” is equivalent to “for all x belongs to S , the negation of $p(x)$ is true”

25. TRUTH VALUE OF PROPOSITIONS WHICH CONTAIN QUANTIFIERS

Whenever we want to find the truth value of a statement consisting quantifier, our approach is according to the following:

1. If the statement consists universal quantifier, then we try to find at least one value belongs to the domain set that makes the statement false. That particular value is the counter example. If there exists a counter example then the statement is false. The statement is true only when there is no counter example e.g. let $p(x)$ be a propositional function whose domain set is A . Let for $x=a$ the statement $(\forall x \in A) p(x)$ is false then ‘ a ’ is a counter example
2. If the statement consists existential quantifier, then we try to find at least one value belongs to the domain set that makes the statement true. If such a value exists then the statement is true. The statement is false only if there is no such value that makes the statement true.

ii. $(\exists x \in B) p(x)$ is also a true statement since

$T_p = \{x/ x \in B, p(x)\} = \{ \text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Khalīd-bin-Waleed} \} \neq \phi$

iii. $(\forall x \in B) q(x)$ is a false statement since

$T_q = \{x/ x \in B, q(x)\} = \{ \text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī} \} \neq B$

where as

$(\exists x \in B) q(x)$ is a true statement since

$T_q = \{x/ x \in B, q(x)\} = \{ \text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī} \} \neq \phi$

$(\forall x \in B) r(x)$ is a false statement since

$T_r = \phi \neq A$

Similarly

$(\exists x \in B) r(x)$ is also a false statement since

$T_r = \phi$

24. NEGATION OF PROPOSITIONS WHICH CONTAIN QUANTIFIERS

Consider the Ayat *Every soul will taste of death (Āl-e-Imrān: 185)* i.e., “every soul is mortal” which is a proposition. The negation of this proposition is “it is not true that every soul is mortal”; in other words, there exists at least one soul who is not mortal. Symbolically, then, if S denotes the set of souls, then the negation of the proposition can be written as:

$\sim(\forall x \in S) (x \text{ is mortal}) \equiv (\exists x \in S)(x \text{ is not mortal})$

which reads “For all” or “For every” is called the universal quantifier. Notice that $(\forall x \in A) p(x)$ or $\forall x, p(x)$ is equivalent to the set theoretic statement that the truth set of $p(x)$ is the entire set A , that is,

$$T_p = \{x/x \in A, p(x)\} = A$$

Similarly

$$(\exists x \in A)p(x) \text{ or simply } \exists x, p(x)$$

is a statement which reads “There exists an element x belongs to set A such that $p(x)$ is a true statement or simply “For some x , $p(x)$ ”.

The symbol

$$\exists$$

which reads “There exists” or “For some” or “For at least one” is called the existential quantifier. Notice that $(\exists x \in A) p(x)$ or $\exists x, p(x)$ is equivalent to the set-theoretic statement that the truth set of $p(x)$ is not empty, that is

$$T_p = \{x/x \in A, p(x)\} \neq \phi$$

Consider

$B = \{ \text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Khalīd-bin-Waleed} \}$

with three propositional functions

- iv. $p(x)$: x is saḥābī
- v. $q(x)$: x is ashra-e-mubashshirah
- vi. $r(x)$: x is died in 20th century

where $x \in B$

once again then

- i. $(\forall x \in B) p(x)$ is a true statement since
 $T_p = \{x/ x \in B, p(x)\} = \{ \text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Khalīd-bin-Waleed} \} = B$

Similarly

$B = \{\text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Khalīd-bin-Walced}\}$

with three propositional functions

- i. $p(x)$: x is saḥābī
- ii. $q(x)$: x is ashra-e-mubashshirah
- iii. $r(x)$: x is died in 20th century

where $x \in B$

Here

$p(x)$ is true for all $x \in B$ i.e. the truth set of $p(x)$ is the set B

$q(x)$ is true for some $x \in B$ and the truth set of $q(x)$ is:

$\{\text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī}\}$

$r(x)$ is false for all $x \in B$ so its truth set is \emptyset

Notice, by the preceding example, that a propositional function defined on a set B could be true for all $x \in B$, for some $x \in B$ or for no $x \in B$

23. QUANTIFIER

Let $p(x)$ be a propositional function on a set A , then

$(\forall x \in A) p(x)$ or simply $\forall x, p(x)$

is a statement which reads “For every element x belongs to set A . $p(x)$ is a true statement”, or simply “For all x , $p(x)$ ”.

The symbol

\forall

- r)] iii. $x = \text{Hazrat Usmān}$
 iv. $x = \text{Hazrat Alī}$
 (since each of the first four is saḥābī)

Where as it is false for

- $x = \text{Hazrat Umar-bin-Abdul Azīz}$
 (since Hazrat Umar-bin-Abdul Azīz is not a saḥābī)

Comparing $p(x)$ with the tree diagram given in section #3, we observe that the sentence is

- i. Clear
 ii. Certain
 iii. Complete
 iv. Mathematical
 v. Open

Moreover, it consists a mathematical variable x therefore $p(x)$ is a propositional function (or an open sentence)

The set A is called the “domain” or “replacement set”.

The set $\{\text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī}\}$ which is the set of all replacements that changes the propositional function into true sentences is called the solution set or the truth set i.e., if $p(x)$ is a propositional function on a set A , then the set of elements $a \in A$ with the property that $p(a)$ is true is called the truth set T_p of $p(x)$.

In other words,

$$T_p = \{x/x \in A, p(x) \text{ is true}\}$$

or, simply

$$T_p = \{x/p(x)\}$$

Notice that a statement consist a truth value (true or false) while a propositional function consists a truth set.

Now consider another set

9. Exportation (Exp.):

$$[(p \wedge q) \rightarrow r] \equiv [p \rightarrow (q \rightarrow r)]$$

10. Tautology (Taut.):

$$p \equiv (p \vee p).$$

$$p \equiv (p \wedge p).$$

21. PLAYFULLNESS

Atheists some time ask the following type questions to confuse muslims.

“If Allah is Omnipotent then whether He can create a stone that He cannot hold?” suppose the question is asked to a person X(not necessarily a muslim). Now suppose according to X, Allah is not Omnipotent, i.e. according to his faith, Allah have not got power over all things and have not absolute control over all affairs. So whatever be the answer of X, it is against the faith of a muslim. But if X is a muslim then according to him, Allah is Omnipotent, i.e. according to him, Allah have power over all things and have absolute control over all affairs. So according to his faith, Allah can create any kind of stone and he can hold any kind of stone. The conjunction of the three statements, i.e. “Allah is Omnipotent and he can create a stone and he can not hold it” is a contradiction. Similarly “Allah is Omnipotent and he cannot create the stone” is also a contradiction. The only true conjunction is “ Allah is Omnipotent and he can create the stone and he can hold it.”

22. PROPOSITIONAL FUNCTION

Let

$\Lambda = \{\text{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Umar bin-Abdul Azīz}\}$

For $x \in \Lambda$, consider the following sentence

$p(x)$: x is saḥābī

Now $p(x)$ is true for

- i. $x = \text{Hazrat Abu Bakr}$
- ii. $x = \text{Hazrat Umar}$

20. THE RULES OF REPLACEMENT

There are many valid truth-functional arguments that cannot be proved valid using only the nine Rules of Inference that have been given thus far. For example, a formal proof of validity for the obviously valid argument

$$\begin{array}{c} p \wedge q \\ \therefore q \end{array}$$

requires additional Rules of Inference.

Now the only compound statements that concern us here are truth-functional compound statements. Hence if any part of a compound statement is replaced by an expression that is logically equivalent to the part replaced, the truth value of the resulting statement is the same as that of the original statement. This is sometimes called the Rule of Replacement and sometimes the Principle of Extensionality.

Any of the following logically equivalent expressions can replace each other wherever they occur:

1. De Morgan's Theorem (De M.): $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$.
 $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$.
2. Commutation (Com.): $(p \vee q) \equiv (q \vee p)$.
 $(p \wedge q) \equiv (q \wedge p)$.
3. Association (Assoc.): $[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$.
 $[p \wedge (q \wedge r)] \equiv [(p \wedge q) \wedge r]$.
4. Distribution (Dist.): $[p \wedge (q \vee r)] \equiv [(p \wedge q) \vee (p \wedge r)]$.
 $[p \vee (q \wedge r)] \equiv [(p \vee q) \wedge (p \vee r)]$.
5. Double Negation (D.N.): $p \equiv \sim\sim p$.
6. Transposition (Trans.): $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$.
7. Material Implication (Impl.): $(p \rightarrow q) \equiv (\sim p \vee q)$.
8. Material Equivalence (Equiv.): $(p \equiv q) \equiv [(p \rightarrow q) \wedge (q \rightarrow p)]$.
 $(p \equiv q) \equiv [(p \wedge q) \vee (\sim p \wedge \sim q)]$.

Rules of Inference

1. *Modus Ponens* (M.P.)

$$\begin{array}{l} p \rightarrow q \\ p \\ \therefore q \end{array}$$

2. *Modus Tollens* (M.T.)

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \therefore \neg p \end{array}$$

3. *Hypothetical Syllogism* (H.S.)
(D.S.)

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \therefore p \rightarrow r \end{array}$$

4. *Disjunctive Syllogism*

$$\begin{array}{l} p \vee q \\ \neg p \\ \therefore q \end{array}$$

5. *Constructive Dilemma* (C.D.)

$$\begin{array}{l} (p \rightarrow q) \wedge (r \rightarrow s) \\ p \vee r \\ \therefore q \vee s \end{array}$$

6. *Absorption* (Abs)

$$\begin{array}{l} p \rightarrow q \\ \therefore p \rightarrow (p \wedge q) \end{array}$$

7. *Simplification* (Simp.)

$$\begin{array}{l} p \wedge q \\ \therefore p \end{array}$$

8. *Conjunction* (Conj.)

$$\begin{array}{l} p \\ q \\ \therefore p \wedge q \end{array}$$

9. *Addition* (Add.)

$$\begin{array}{l} p \\ \therefore p \vee q \end{array}$$

These nine Rules on Inference are elementary valid argument forms, whose validity is easily established by truth tables. They can be used to construct formal proofs of validity for a wide range of more complicated arguments. The names listed are standard for the most part, and the use of their abbreviations permits formal proofs to be set down with a minimum of writing.

19. THE RULES OF INFERENCE

A *formal proof of validity* for a given argument is defined to be a sequence of statements, each of which is either a premise of that argument or follows from preceding statements by an elementary valid argument, and such that the last statement in the sequence is the conclusion of the argument whose validity is being proved. This definition must be completed and made definite by specifying what is to count as an 'elementary valid argument'. We first define an *elementary valid argument* as any argument that is a substitution instance of an elementary valid argument form. Then, we present a list of just nine argument forms that are sufficiently obvious to be regarded as elementary valid argument forms and accepted as Rules of Inference.

One matter to be emphasized is that any substitution instance of an elementary valid argument form is an elementary valid argument. Thus the argument

$$\begin{array}{l} \sim r \rightarrow (s \rightarrow t) \\ \sim r \\ \therefore s \rightarrow t \end{array}$$

is an elementary valid argument because it is a substitution instance of the elementary valid argument form *Modus Ponens* (M.P.). It results from

$$\begin{array}{l} p \rightarrow q \\ p \\ \therefore q \end{array}$$

by substituting $\sim r$ for p and $s \rightarrow t$ for q ; therefore, it is of that form even though *Modus Ponens* is not the *specific form* of the given argument.

Following is a list of nine elementary valid argument forms that can be used in constructing formal proofs of validity:

$$p \wedge q \vdash p$$

is known as *Simplification* (Simp)

Argument #12

Five times prayer is obligatory on a muslim

Fasting during the month of Ramazan is obligatory on a muslim

Therefore five times prayer is obligatory on a muslim and fasting during the month of Ramazan is obligatory on a muslim

Symbolically

$$p, q \vdash p \wedge q$$

Where

p: Five times prayer is obligatory on a muslim

q: Fasting during the month of Ramazan is obligatory on a muslim

The argument of the type

$$p, q \vdash p \wedge q$$

is known as *Conjunction* (Conj)

Argument #13

Hazrat Abu Bakr is muslim's Caliph

Therefore Hazrat Abu Bakr is muslim's Caliph or he was the governor of Iraq

Symbolically

$$p \vdash p \vee q$$

Where

p: Hazrat Abu Bakr is muslim's Caliph

q: Hazrat Abu Bakr was the governor of Iraq

The argument of the type

$$p, q \vdash p \vee q$$

is known as *Addition*(Add)